

# Motion Analysis Methods:

## Sampling, Fourier Analysis and Filtering



Engr325

Lab #7

Winter, 2019

### Video Analysis and Filtering

Name \_\_\_\_\_

Partner (s) \_\_\_\_\_ Grade \_\_\_\_/10

#### Introduction

The goal of this lab is to obtain a position-time dataset from video analysis and create a Butterworth filter with appropriate smoothing to obtain useful velocity and acceleration graphs.

#### Objectives

- Understand the basics of video digitization;
- Obtain useful 2D data from a video clip;
- Understand how digital filtering can increase the quality of results.

#### Equipment Provided

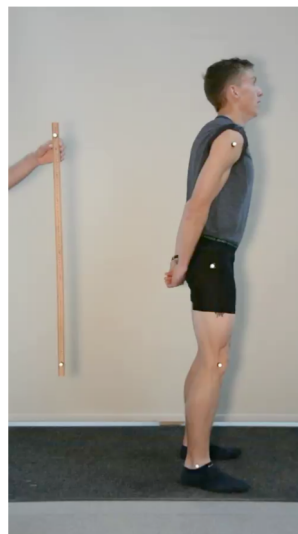
- Computer with appropriate software.

#### References

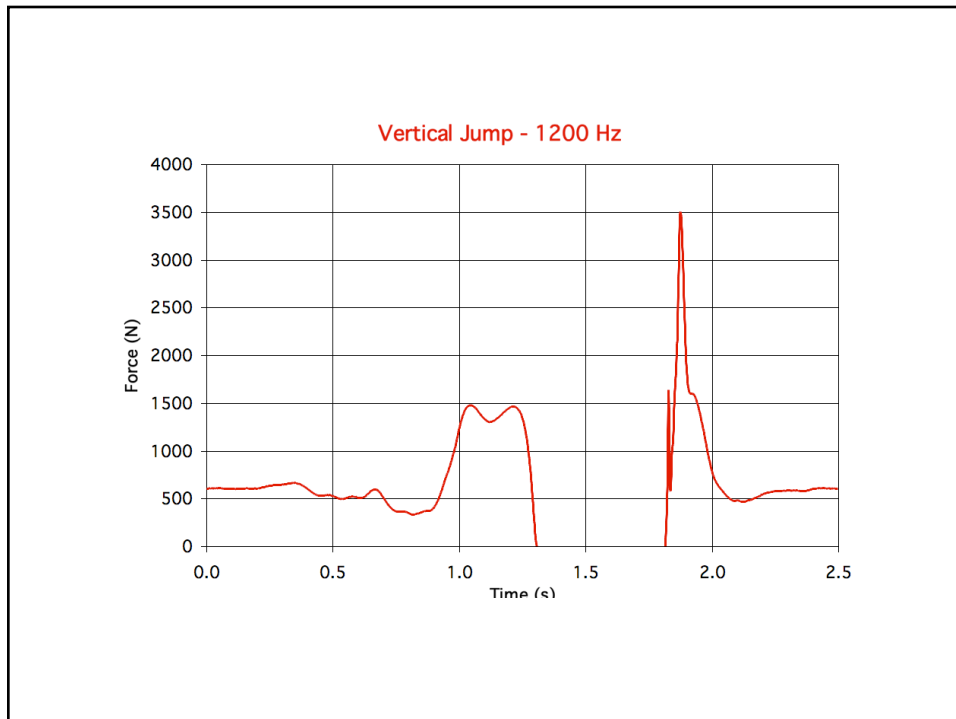
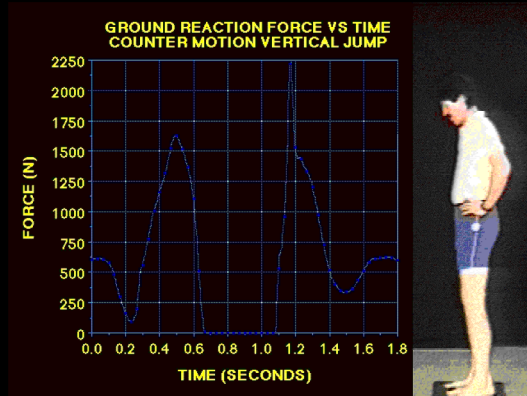
- Video digitization and digital filtering textbooks and web sites.

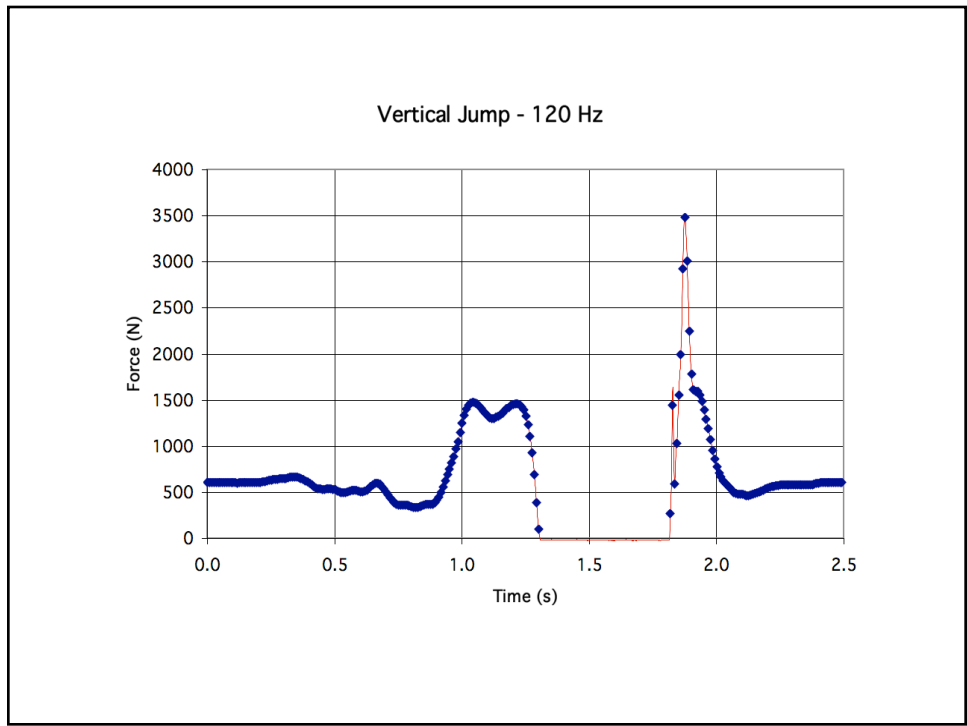
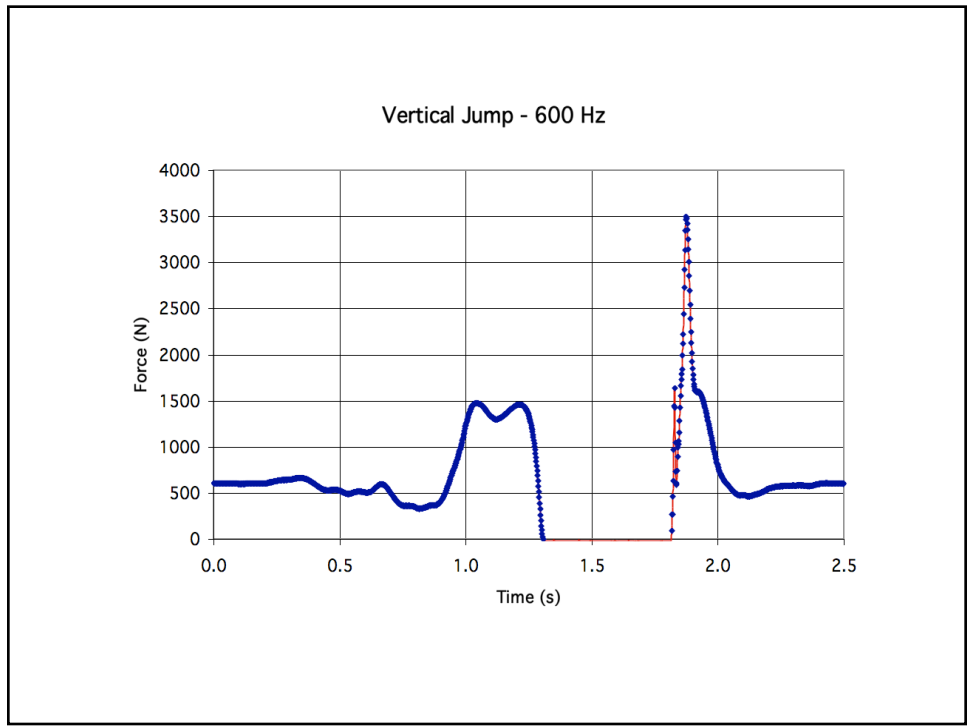
#### Procedure

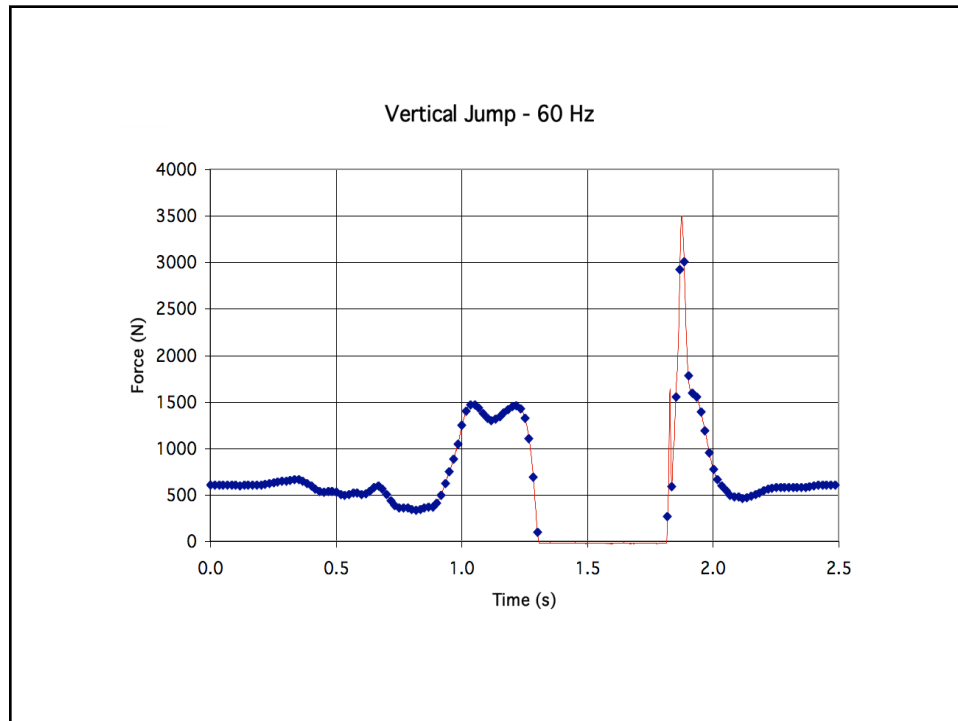
- 1) On the class website under the heading of Lab #7, you will find a video file showing a side view of a human vertical jump with markers indicating various joint centers, including at the hip. Two formats are provided, .mp4 and .mov. Download the .mp4 video file and save it to your lab computer (sometimes the .mov format works better, but see how your results are with the .mp4 file).



# Motion Analysis Methods: Sampling, Fourier Analysis and Filtering



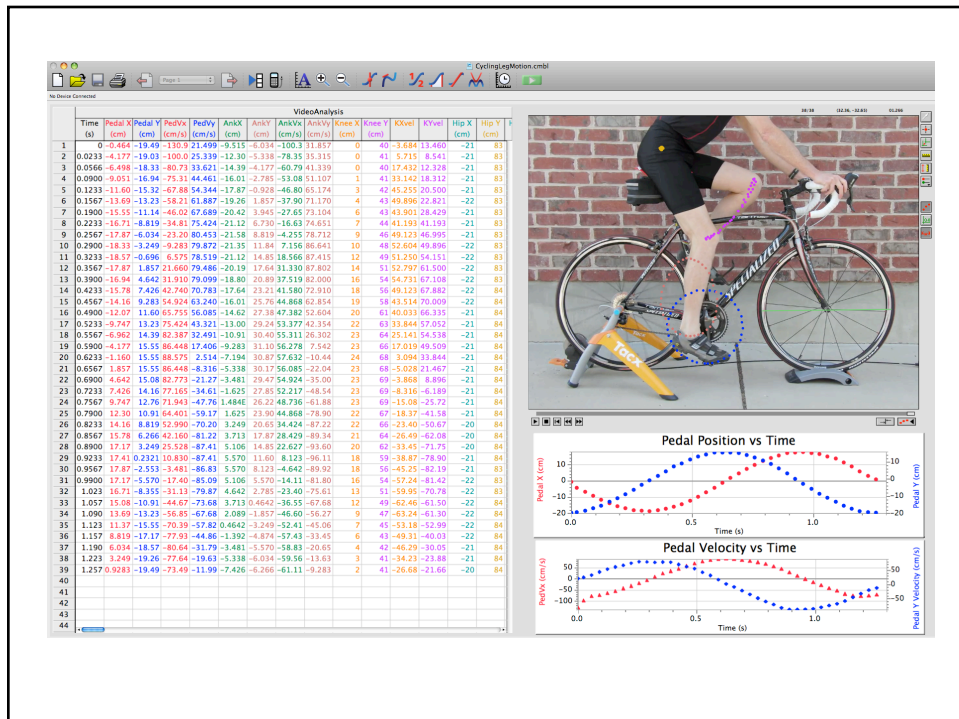




## Proper Sampling:

"If you can exactly *reconstruct* the analog signal from the samples, you must have done the sampling properly."

(from *Digital Signal Processing* by Steven W. Smith, p. 39)

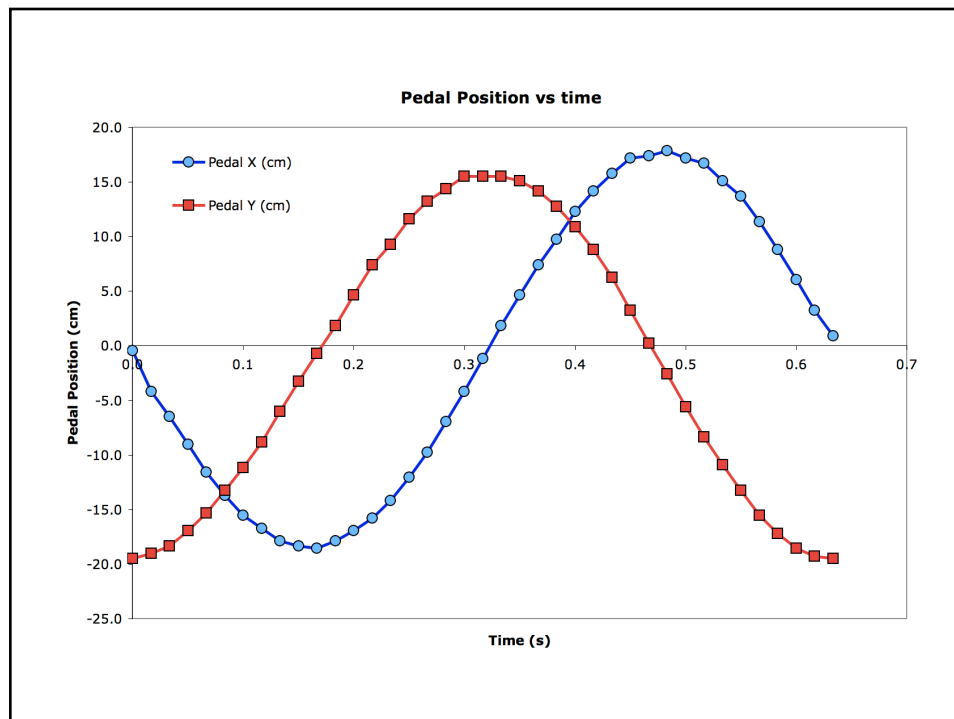


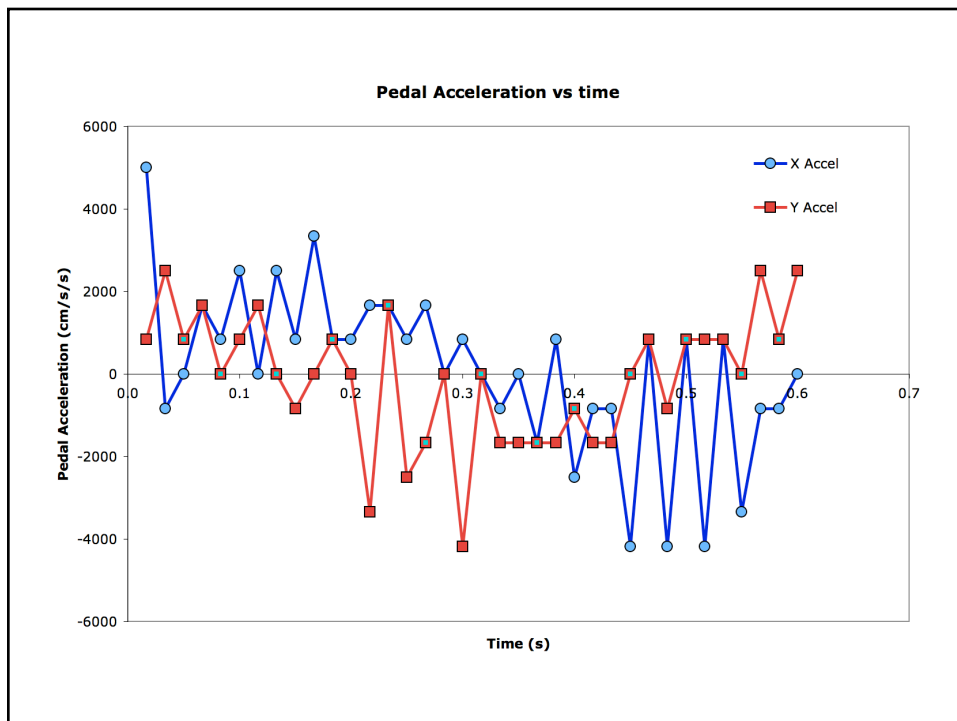
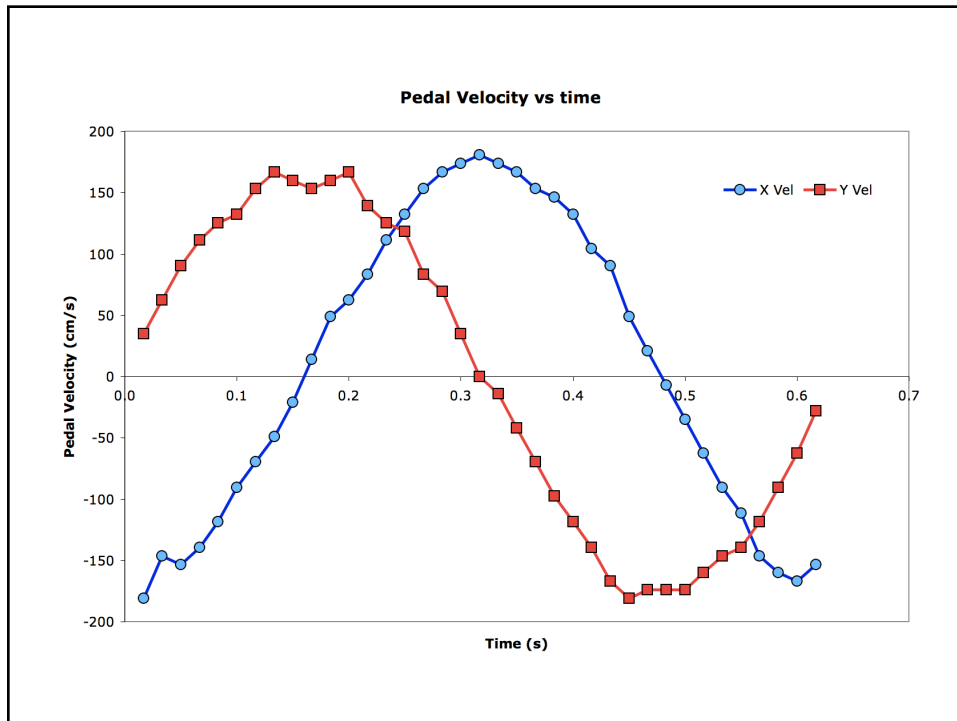
## Analysis Methods for Position-Time Data:

First Central Difference formulas:

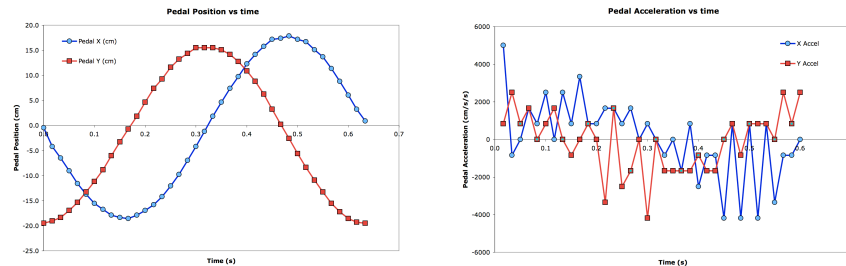
$$\text{Velocity: } V_i = \frac{X_{i+1} - X_{i-1}}{t_{i+1} - t_{i-1}}$$

$$\text{Acceleration: } a_i = \frac{X_{i+1} - 2X_i + X_{i-1}}{(t_{i+1} - t_{i-1})^2}$$





## Why does taking the derivative of position-time data seem to amplify the noise in a signal?



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Low Frequency Signal ( $f = 1 \text{ Hz}$ )

$$f(t) = 100 \sin(2 \pi f t)$$

$$f'(t) = 200 \pi f \cos(2 \pi f t)$$

$$f''(t) = -400 \pi^2 f^2 \sin(2 \pi f t)$$

High Frequency Noise ( $f = 10 \text{ Hz}$ )

$$f_n(t) = 1 \sin(2 \pi f t)$$

$$f'_n(t) = 2 \pi f \cos(2 \pi f t)$$

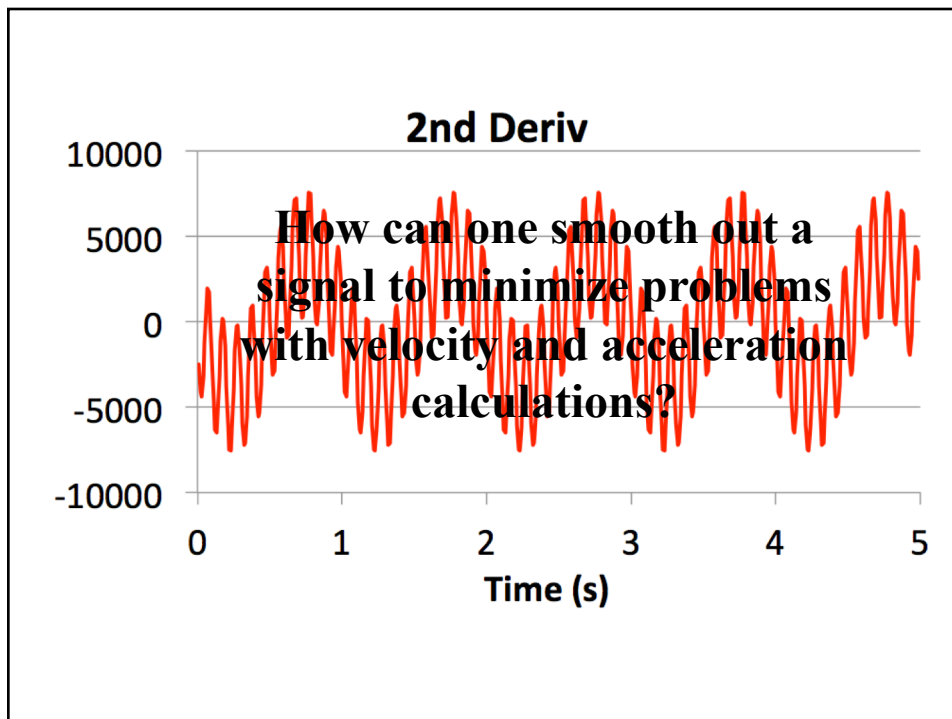
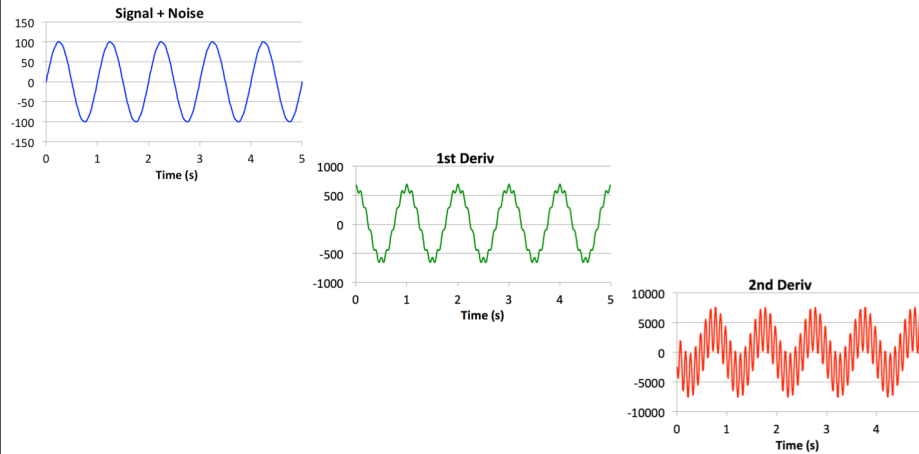
$$f''_n(t) = -4 \pi^2 f^2 \sin(2 \pi f t)$$

Signal to Noise Ratio:

100 : 1
10 : 1
1 : 1



**Why does taking the derivative of position-time data seem to amplify the noise in a signal?**



Butterworth filtering:

$$X'_i = a_0 X_i + a_1 X_{i-1} + a_2 X_{i-2} + b_1 X'_{i-1} + b_2 X'_{i-2}$$

$X$  = raw data points

$X'$  = previously filtered data points (recursion)

$$\omega_c = \tan\left(\frac{\pi f_c}{f_s}\right)$$

$K_1 = \sqrt{2}\omega_c$  for a Butterworth filter,

or,  $2\omega_c$  for a critically damped filter

$$K_2 = \omega_c^2, \quad a_0 = \frac{K_2}{(1 + K_1 + K_2)} \quad a_1 = 2a_0, \quad a_2 = a_0$$

$$K_3 = \frac{2a_0}{K_2}, \quad b_1 = -2a_0 + K_3$$

$$b_2 = 1 - 2a_0 - K_3, \quad \text{or,} \quad b_2 = 1 - a_0 - a_1 - a_2 - b_1$$

**Table 2.2 Coefficients for Butterworth Low-Pass Filter**

$f_s/f_c$	$a_0$	$a_1$	$a_2$	$b_1$	$b_2$
4	0.29289	0.58579	0.29289	0.00000	-0.1715 <i>i</i>
5	0.20657	0.41314	0.20657	0.36953	-0.19582
6	0.15505	0.31010	0.15505	0.62021	-0.24041
7	0.12123	0.24247	0.12123	0.80303	-0.28796
8	0.09763	0.19526	0.09763	0.94281	-0.33333
9	0.08042	0.16085	0.08042	1.05333	-0.37502
10	0.06746	0.13491	0.06746	1.14298	-0.41280
11	0.05742	0.11484	0.05742	1.21719	-0.44687
12	0.04949	0.09898	0.04949	1.27963	-0.47759
13	0.04311	0.08621	0.04311	1.33291	-0.50533
14	0.03789	0.07578	0.03789	1.37889	-0.53045
15	0.03357	0.06714	0.03357	1.41898	-0.55327
16	0.02995	0.05991	0.02995	1.45424	-0.57406
17	0.02689	0.05379	0.02689	1.48550	-0.59307
18	0.02428	0.04856	0.02428	1.51338	-0.61051
19	0.02203	0.04407	0.02203	1.53842	-0.62655
20	0.02008	0.04017	0.02008	1.56102	-0.64135
21	0.01838	0.03676	0.01838	1.58152	-0.65505
22	0.01689	0.03378	0.01689	1.60020	-0.66776
23	0.01557	0.03114	0.01557	1.61730	-0.67958
24	0.01440	0.02880	0.01440	1.63299	-0.69060
25	0.01336	0.02672	0.01336	1.64746	-0.70090

Butterworth Low-Pass Filter is easy to implement in Excel:

	A	B	C	D	E	F	G
1	<b>Butterworth Filter Coefficients: Vertical Jump Filtering</b>						
2						<b>A0</b>	0.06746
3				<b>Wc</b>	0.32492	<b>A1</b>	0.13491
4	<b>SAMPLING FREQUENCY:</b>	<b>60</b>	<b>K1</b>	0.45951		<b>A2</b>	0.06746
5	<b>CUTOFF FREQUENCY:</b>	<b>6</b>	<b>K2</b>	0.10557		<b>B1</b>	1.14298
6			<b>K3</b>	1.27789		<b>B2</b>	-0.41280

## How to do simple 2D motion analysis?

The screenshot shows the LoggerPro interface for video analysis. On the left, a 'VideoAnalysis' table tracks the position of a point on the video over time. The table has columns for Time (s), X (cm), and Y (cm). The video frame on the right shows a cyclist on a Tacx stationary bike with blue dots marking the center of the wheels, the pedal, and the rider's hands and feet. A large 'LoggerPro' watermark is overlaid on the video.

Time (s)	X (cm)	Y (cm)
0	15.58	-18.23
0.03000	12.35	-20.29
0.06000	9.704	-22.06
0.09000	6.469	-23.23
0.1200	2.941	-24.41
0.1500	-0.8822	-24.41
0.1800	-4.705	-23.82
0.2100	-7.645	-22.65
0.2400	-10.88	-21.76
0.2700	-14.11	-19.41
0.3000	-16.17	-17.06
0.3300	-18.53	-14.41
0.3600	-19.70	-11.76
0.3900	-20.88	-8.823
0.4200	-21.47	-5.588
0.4500	-21.76	-0.59
0.4800	-21.17	176
0.5100	-20.00	11
0.5400	-18.82	16
0.5700	-16.47	18
0.6000	-14.11	18
0.6300	-11.17	14.70
0.6600	-7.939	16.18
0.6900	-4.411	17.65
0.7200	-0.8822	17.94
0.7500	2.646	17.94
0.7800	6.175	17.06
0.8100	9.704	16.18
0.8400	12.64	14.41
0.8700	15.00	11.76
0.9000	17.35	9.411
0.9300	19.11	6.470
0.9600	20.58	2.647



### LoggerPro Software can be used to analyze video images:

The screenshot displays the LoggerPro software interface for video analysis. On the left, a data table tracks the position of a point on the wheel over time. The central window shows a video frame of a wheel with a yellow crosshair and a green line indicating a point of interest. On the right, two graphs are shown: 'X and Y Position vs Time' and '2D Motion Graph'.

Time (s)	X (cm)	Y (cm)	
6	0.1500	25.48	-19.89
7	0.1800	27.80	-16.62
8	0.2100	29.43	-13.08
9	0.2400	30.93	-9.402
10	0.2700	31.89	-5.450
11	0.3000	32.29	-1.499
12	0.3300	32.16	2.316
13	0.3600	31.63	6.132
14	0.3900	30.52	10.08
15	0.4200	29.02	13.63
16	0.4500	26.98	17.03
17	0.4800	24.80	20.17
18	0.5100	22.07	23.16
19	0.5400	18.80	25.62
20	0.5700	15.53	27.66
21	0.6000	11.86	29.43
22	0.6300	8.176	30.66
23	0.6600	4.088	31.14
24	0.6900	0.2725	31.75
25	0.7200	-3.815	31.48
26	0.7500	-7.903	30.52
27	0.7800	-11.72	29.30
28	0.8100	-15.53	27.39
29	0.8400	-18.94	25.21
30	0.8700	-22.07	22.48
31	0.9000	-24.96	19.48
32	0.9300	-27.25	16.08
33	0.9600	-29.16	12.26
34	0.9900	-30.66	8.175
35	1.020	-31.48	4.024
36	1.050	-31.75	-0.136
37	1.080	-31.48	-4.360
38	1.110	-30.66	-8.448
39	1.140	-29.30	-12.54
40	1.170	-27.66	-16.21
41	1.200	-25.21	-19.89
42	1.230	-22.35	-23.03
43	1.260	-19.35	-25.75
44	1.290	-15.81	-28.07
45	1.320	-12.13	-29.84
46	1.350	-8.312	-31.34
47	1.380	-4.360	-32.16
48	1.410	-0.136	-32.29
49	1.440	3.952	-32.16